[Project B] Maths Analysis Document: *Loot*

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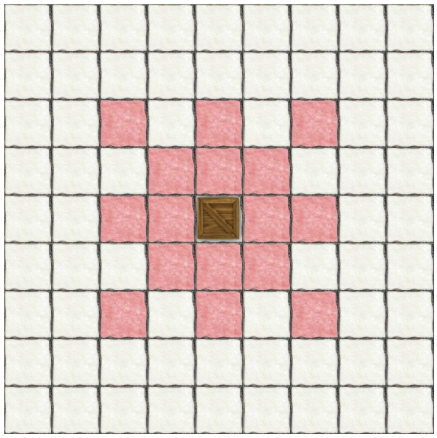
The Space & Affordances

Loot is a strategy game with the most atom-sized degree of resource management and turn complexity. As such, there is not much "hardcore" math analysis to do here, for example compared to other strategy boardgames like *Settlers of Catan*. Indeed, the only thing this document can really analyze mathematically is the game space itself, and the affordances players have with that space. You cannot use the same kind of math to justify *Loot* as you might *Catan*, but the math for Chess or Go would work great here.

In the final version of *Loot*, the space is simply a 9x9 inch grid. The primary affordance we are dealing with in terms of space is that players can move pawns one or two consecutive tiles diagonally, vertically, or horizontally, and pawns must land on enemies to take them. As a reminder from the rulebook, movement looks like the image at right:

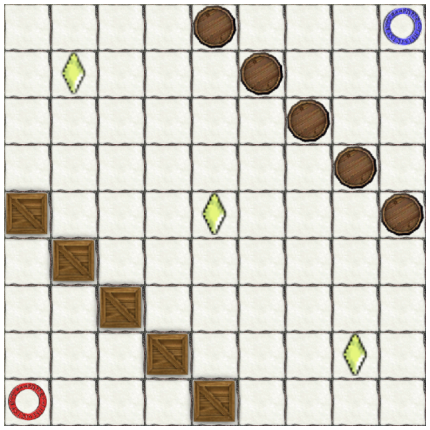


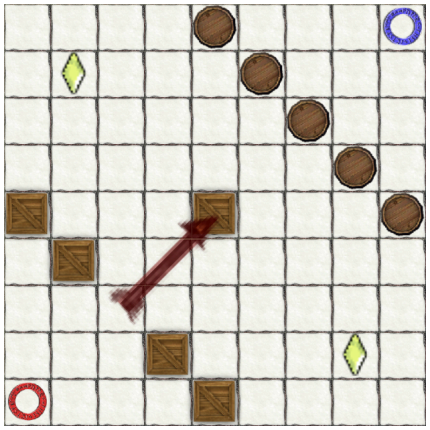
Some math figures, such as the minimum number of turns, average time per turn, length of the game, and so on, do not matter so much in terms of this space. Sure, the minimum number of turns is 7, and the average time per turn in playtesting is around 25 seconds, the average game lasts 11 minutes… but these are mere top-level measurements, and do not inform any bottom-up analyses of design. Again, to justify the present design mathematically, I will be focusing on the physical space of the board, and the rules of motion within that space afforded to players.

Observation 1: The first moves are sacrificial.

The Math

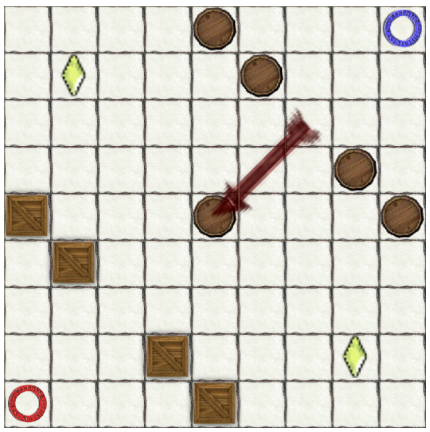
Consider the board and the following first move:



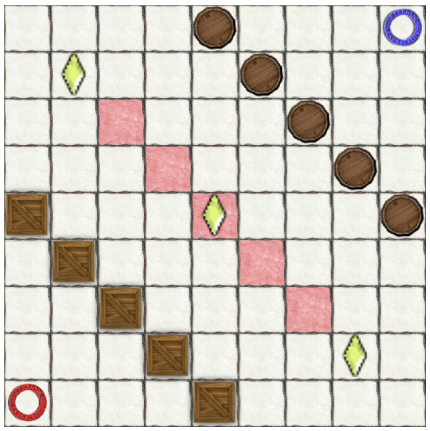


To get to their goal in the most direct route, a player might be tempted to move like the above.

However, a savvy player would never do such a thing on the first move, lest **their pawn gets immediately taken:**



Indeed, we find that **there is a dangerous no-man's-land** running through the center of the board:



The red tiles in the image above show where players are not likely to ever move on the first turn; doing so would be suicide. The player is left with two options: try to go around this red zone, or try to set it up so that if they put a pawn in one of those tiles, they can immediately take whichever enemy pawn would take their pawn. This is the first iota of strategy learned by players—in playtesting, *all* testers recognized this dynamic at the start of their first game.

The Why

This whole situation is an intentional design. The goal is to urge players to think harder on their very first turns, instead of filling their early game with complex setups that might be overwhelming for casual or first-time players. The design here is to present players with an obvious problem, the best solution to which is the *opposite* of obvious (if one even exists). If the board were bigger, meaning moving directly toward the enemy flag would not put a pawn in the red zone, then the best first move *would* be obvious: just move forward, and figure out the rest later. Instead, with this 9x9 board, players' first moves are likely going to be setups for protecting pawns as they make their way toward their goal with more imperfect heuristics. Of course, as the enemy does the same, new openings will arise, old openings will close, and heuristics will adjust accordingly. This is very much like the way time-tested games such as Chess or Go work, in that you can have an initial plan, but it quickly changes based on openings your opponent creates. The main difference in *Loot*, is primarily that my design seeks to distill this dynamic, and coerce it into occurring much sooner in the turn history.

Even as the first few moves take some serious thought, there must come a point where a player makes a sacrificial move. (Else, it happens by accident and a player puts a pawn in peril when they did not mean to). This sacrificial move is an important turning point in the game (you will find it indicated in all of my playtest notes). After the first pawn is taken, heuristics begin to come to fruition (or they fail), and a new mid-term heuristic needs to be formed. This is the primary example of how the game space is the most direct knob we have on the timing and rhythm of player heuristics in *Loot*.

Observation 2: The Respawn Tiles

The Math

There are 3 respawn tiles. They are along the center diagonal of the board, they have 2 tiles between each other, and they are collinear with the red zone discussed in Observation 1.

The Why

I have explored the process of arriving here in the Concept and Design Document in great depth, but I will consolidate the results of this process here.

Why the centerline?

+ It is good to reset the player at a neutral position, where whether to attack, to go for a loot tile, or to defend are each a valid choice, depending.
+ Respawning a pawn at points along the centerline can be used to implement defensive zoning heuristics, without which a player might be at a runaway disadvantage.
+ It's intuitively balanced, both spatially and otherwise.

Why are there 3 respawn tiles?

At first, there were 8. Then, there were 4, then 6, and now 3. So first off, what was wrong with 8 respawn tiles?

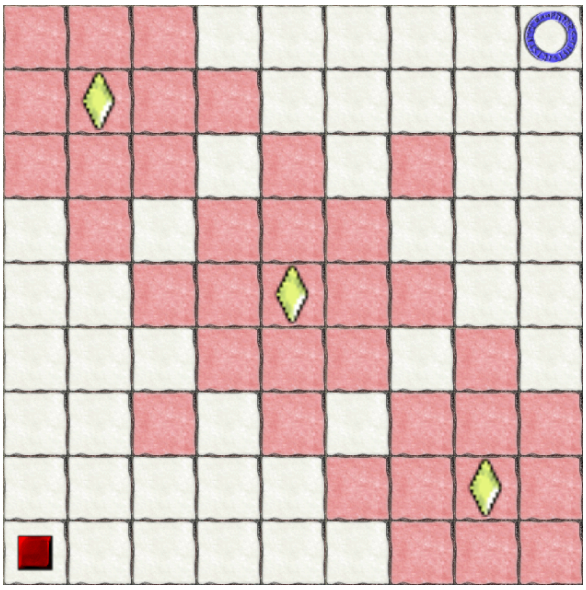
8 respawn tiles meant there was pretty much *always* an open respawn tile. The affordance of always being able to respawn, along with the fact that there were so many of them, meant that skirmishes were becoming very concentrated and very long-winded at certain points. This was the main problem, but I also wanted blocking off *all* respawn tiles to be an affordable strategy for players, which they would not be able to do if there were 8 tiles and 5 pawns.

Next: Why not 4 respawn tiles? At the time, I was trying to declutter the center of the board, as being able to respawn in numerous places was causing the aforementioned long-winded skirmishes. They were long-winded because players could keep respawning over and over again, which felt like an unintentional short-circuit in the loop design. 4 tiles *almost* seemed to alleviate this, but it wasn't quite enough. Not to mention, even though the "respawn blockade" heuristic was possible with 4 tiles, with 5 pawns per player it didn't attract much attention.

Turns out that all issues I noted with regards to the respawn mechanic were alleviated after I tweaked the number to 3 in iteration 6. But—mathematically—*why* did this work so well?

Why are there 2 spaces between respawn tiles?

This is actually very important. Dodge this, I'm about to throw another diagram at you:



This of course represents the movement/attack options of pawns if they are all on respawn tiles. Do you notice the two white diagonal "bridges" connecting the two sides? (You might already be able to see why they're important.) If the board were smaller, or if the tiles were closer together, then these bridges would not exist. Recall the "respawn blockade" gambit: a player ensures that each respawn tile is covered by a pawn, disallowing their opponent to respawn *and* contributing to a blockade along the centerline.

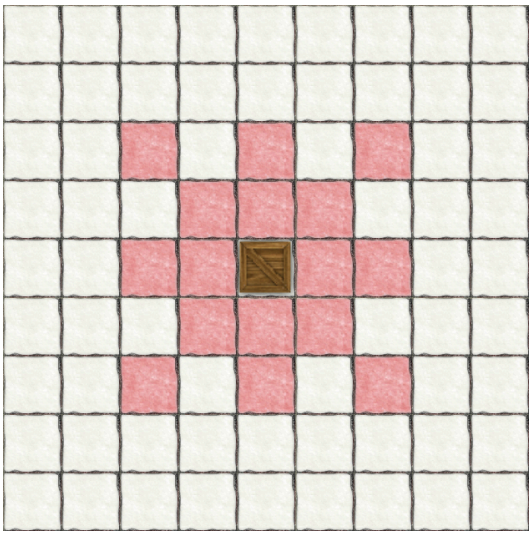
The bridges are important because it allows a player to maneuver around such blockades, which in turn forces their enemy to make movements if they want to maintain their defense. By moving a pawn off of a respawn tile in order to intercept their enemy, the player who holds the centerline has now allowed their enemy to respawn where they once were. At the same time, a pawn on another respawn tile would not be able to take out that newly-spawned pawn *because*

there are 2 tiles between them, not 1. So, the design goal in this placement is to mainly give players opportunities to cleverly get around a total respawn blockade, or else hold disruptive positions in the middle of it. There are two major costs to this permitted heuristic, of course:

(1) Utilizing a bridge requires specific maneuvering, which may take extra turns or betray the heuristic to the blockading opponent. **The currency here is entropy.**

(2) The bridges are hidden information to most beginners, even though a larger portion of beginners discover respawn blockading. **The currency here is experience.**

Observation 3: Pawns can move up to 2 tiles.



The Why

The previous two observations were highly dependent on this rule. So, was it an arbitrary selection? No, not really. My process for deciding on the two-tile movement comes down to a series of questions.

Why not 1 tile?

(1) Movement would be slow, the game would be made magnitudes longer.

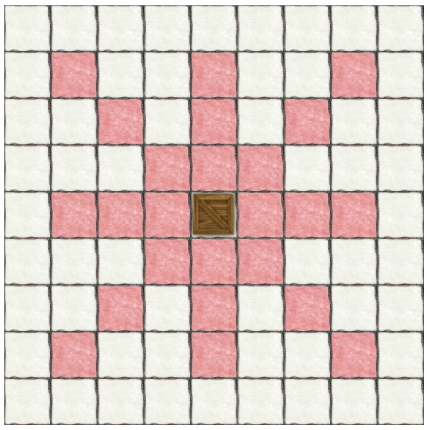
(2) Getting in range to take out a pawn would be difficult, while avoiding it would be easy if not for the extra turns (valuable time) it would take.

(3) **It would make it hard to deceive your opponent.**

I really just want to talk about point (3) from above. A playtester of *Loot* once said something along the lines of, "Games like these harp on player deception. Many things are decided based on whether you or your opponent noticed somebody fuck up or not."

An astute observation, I think. Much of the time, a pawn will be taken out because a player moved it to a spot that they did not initially see as dangerous. If the movement range is merely one tile, however, it is so much easier to tell whether or not a pawn is within range of another. Honestly, I can't imagine someone who *wouldn't* be able to see it, if they were right next to each other. Thus, one tile movement would not be viable.

Why not 3 tiles?



(1) The space is too small.

(2) The option to move 1 or 2 or 3 spaces every turn adds unneeded decision complexity.

(3) Pawns would be dropping left and right, many due to too many half-baked strategies.

(4) It invalidates the respawn tile configuration. Again.

I do not want to spend too much time or paper entertaining this idea. Reason (1) sums it up best: the space is simply too small. Two tiles is best.

Observation 4: Each player has 5 pawns.

Let's assume every observation up to this point is already mathematically justified, including the additional observation that *Loot*'s gameboard is a 9x9 grid space, and do some calculations.

Average Movement Potential ("Reach") and Zone of Contest

A pawn's movement potential (including the tile they occupy, or movement = 0) changes depending on where it is on the board. I've determined there are 6 base cases for this:

Case F: 7

1	1	1						
1	1							
1		1						

Case C: 12

1	1	1						
1	1	1	1					
1	1	1						
	1		1					

Case E: 9

1	1	1	1					
1	1	1						
	1		1					

Case B: 14

	1	1	1					
1	1	1	1	1				
	1	1	1					
1		1		1				

Case D: 11

1	1	1	1	1				
	1	1	1					
1		1		1				

Case A: 17

1		1		1				
	1	1	1					
1	1	1	1	1				
	1	1	1					
1		1		1				

These cases are copied and rotated around the board. This arrangement can be represented like this:

F	E	D	D	D	D	D	E	F
E	C	B	B	B	B	B	C	E
D	B	A	A	A	A	A	B	D
D	B	A	A	A	A	A	B	D
D	B	A	A	A	A	A	B	D
D	B	A	A	A	A	A	B	D
D	B	A	A	A	A	A	B	D
E	C	B	B	B	B	B	C	E
F	E	D	D	D	D	D	E	F

	Count	Sum
A	25	425
B	20	280
C	4	48
D	20	220
E	8	72
F	4	28
	81	1073
	Average:	**13.2**
	(x5)	66.2
	(% of board)	**82%**
	(% contested)	**67%**

Note the bolded calculations. These say that the average pawn (ignorant of where exactly it is on the board) has a "reach" area of around 13 tiles. A team of 5 pawns, then, has a maximum expected reach of about 82% of the board. **This is for one player.** If we consider the reach of two players, we can calculate the maximum expected "contested" area simply by multiplying them. With 5 pawns each, this max contested area is 67%, roughly two-thirds of the board.

I believe this is the sweet spot, right in the goldilocks zone. Just one more pawn, and suddenly the max expected reach becomes 98% for each player, and the zone of contest 96%. In a game where all pieces move and attack the same way, it's *critical* to get these ratios of territories correct; I know from playtesting that having nearly 100% of the board be contested would lead to a lot of boring heuristics and unsatisfying stalemates. Maintaining the maximum zone of contest at 67% still leaves—at *worst*—a third of the board to maneuver in and form more interesting strategies.